In the simulation of turbulent flow, it is imperative that interpolations within a discrete array of elements, whether they represent fluid density, components of velocity, etc. are precise. Various polynomial fits, whether linear, quadratic, or higher order can repeat across three dimensions to obtain a corresponding three-dimensional interpolation. For the purposes of this discussion, the trilinear interpolation will be considered a baseline to compare accuracy to.

We will consider a coordinate grid with dimensions , and . Additionally, . As a result, the process to obtain an interpolation at a given position is as follows:

* At each index, interpolate at for each index.
* At each index, interpolate at within the previously interpolated values.
* Interpolate at within the previously interpolated values

It becomes apparent that there are possible interpolation orderings, though this was shown to have negligible effect on interpolation results as seen in Figure (1) below.

A one-dimensional discrete Fourier Interpolation can be evaluated through the following formula:

where is the complex Fourier term evaluated at each position within domain . Equation (1) makes up the backend of libraries such as FFTW, or numpy.fft. The reconstruction of the Fourier terms can be evaluated at the desired position as follows:

However, this formula has several major assumptions:

* The discrete sequence must be -periodic. Any periodic discontinuities will nullify the advantages a Fourier Interpolation has over Linear Interpolation
* To avoid Gibbs Phenomenon, the discrete sequence must converge to a continuously differentiable function as the number of elements within the sequence increases towards infinity.

In comparison to the Fourier Interpolation, a Linear Interpolation is trivial.

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Figure 1: *Interpolation Error for each Ordering*